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MATDIP301

Third Semester B.E. Degree Examination, June/July 2013

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find modulus and amplitude of $1+\cos\theta+i\sin\theta$. **(06 Marks)**

b. If n is positive integer, prove that $(\sqrt{3}+i)^n+(\sqrt{3}-i)^n=2^{n+1}\cos\left(\frac{n\pi}{2}\right)$. **(07 Marks)**

c. Find the cube root of $1+i$ and represent them in the Argand diagram. **(07 Marks)**

2 a. Find the n^{th} derivative of $e^{ax}\sin(bx+c)$. **(06 Marks)**

b. If $y=e^{m\cos^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$. **(07 Marks)**

c. Find the n^{th} derivative of $\frac{x^2}{(x+2)(2x+3)}$. **(07 Marks)**

3 a. Prove that $\tan\phi=r\frac{d\theta}{dr}$ with usual notations. **(06 Marks)**

b. Find the pedal equation for the curve $r=a(1+\cos\theta)$. **(07 Marks)**

c. Expand $f(x)=\sqrt{1+\sin 2x}$ using Maclaurin's series upto 4^{th} term. **(07 Marks)**

4 a. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$. **(06 Marks)**

b. If $u=f(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. **(07 Marks)**

c. If $u=\tan^{-1}x+\tan^{-1}y$ and $V=\frac{x+y}{1-xy}$, find the value of $\frac{\partial(u,v)}{\partial(x,y)}$. **(07 Marks)**

5 a. Obtain the reduction formula for $\int\cos^n x\,dx$ where n is a positive integer. **(06 Marks)**

b. Evaluate $\int_0^2 x^{5/2}\sqrt{2-x}\,dx$. **(07 Marks)**

c. Evaluate $\int_1^2\int_3^4(xy+e^y)dydx$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Evaluate $\int_0^1\int_0^1\int_0^1 e^{x+y+z}\,dxdydz$. **(06 Marks)**

b. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$. **(07 Marks)**

c. Show that $\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta \times \int_0^{\pi/2}\frac{1}{\sqrt{\sin\theta}}\,d\theta=\pi$ **(07 Marks)**

7 a. Solve $xy\frac{dy}{dx}=1+x+y+xy$. **(06 Marks)**

b. Solve $\left[x\tan\left(\frac{y}{x}\right)-y\sec^2\left(\frac{y}{x}\right)\right]dx+x\sec^2\left(\frac{y}{x}\right)dy=0$ **(07 Marks)**

c. Solve $\frac{dy}{dx}+y\cot x=4x\,\mathrm{cosec}\,x$. **(07 Marks)**

8 a. Solve $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=2e^{3x}$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=\sin 2x$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2}+4y=1+x^2$ **(07 Marks)**

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